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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.
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<tr>
<td>1</td>
<td>Equate impulse to momentum to find initial speed ( v ) and Newton’s law of restitution to find new speed: ( v = 4u ), ( v' = ev = [-] 3u )</td>
<td>M1 A1</td>
<td>2</td>
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<tr>
<td>2</td>
<td>Find ( v^2 ) at both ( A ) and ( B ): ( v_A^2 = \omega^2(a^2 - 0.5^2) ) and ( v_B^2 = \omega^2(a^2 - 0.75^2) )</td>
<td>B1</td>
<td></td>
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<tr>
<td></td>
<td>Find amplitude ( a ) m from given K.E. ratio: ( \frac{1}{2} mv_A^2 = (12/11) \frac{1}{2} mv_B^2 ) ( 11(a^2 - 0.5^2) = 12(a^2 - 0.75^2) ) ( a^2 = \frac{1}{4}(27 - 11) = 4 ), ( a = 2 )</td>
<td>M1 A1</td>
<td>3</td>
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<tr>
<td></td>
<td>Find ( \omega ) from ( v_{\text{max}} = a\omega ) ( 0.6 = 2\omega ), ( \omega = 0.3 )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Find time (( \phi ) on ( a )) at ( A ) ( \omega^{-1}\sin^{-1}(0.5/2) ) or ( \omega^{-1}\cos^{-1}(0.5/2) )</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>or at ( B ), e.g.: ( \omega^{-1}\sin^{-1}(0.75/2) ) or ( \omega^{-1}\cos^{-1}(0.75/2) )</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Combine correctly to find time from ( A ) to ( B ): ( \omega^{-1}\sin^{-1}(0.75/2) - \omega^{-1}\sin^{-1}(0.5/2) ) ( \text{or } \omega^{-1}\cos^{-1}(0.5/2) - \omega^{-1}\cos^{-1}(0.75/2) )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Evaluate to 3 d.p.: ( = \omega^{-1}(0.3844 - 0.2527) ) ( \text{or } \omega^{-1}(1.318 - 1.186) ) ( = 1.2813 - 0.8423 ) ( 4.3937 - 3.9547 = 0.439 ) [s]</td>
<td>A1</td>
<td>5</td>
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3. Use conservation of momentum, e.g.:

\[ mv_A + 9mv_B = mu \]

M1

Use Newton’s law of restitution (consistent signs):

\[ v_B - v_A = eu \]

M1

Relate \( v_A \) to \( v_B \) using K.E. (A.E.F.):

\[ \frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{4}mu^2 \]

M1

Combine two eqns to find \( v_A \) and \( v_B \) e.g.:

\[ v_A = (1 - 9e)u/10, \quad v_B = (1 + e)u/10 \]

or \( v_A, v_B = -u/2, u/6 \) [or 7u/10, u/30]

M1 A1

Use in 3rd eqn to find \( e \), e.g.:

\[ (1 - 9e)^2 + 9(1 + e)^2 = 50 \]

(A0 if finally ±\( \frac{7}{5} \))

90 \( e^2 \) = 40, \( e = \frac{\sqrt{5}}{5} \)

M1 A1 7

Use Newton’s law of restitution with

\[ v_C = 2v_B', \text{ e.g.: } v_C - v_B' = ev_B, \quad v_B' = \frac{5}{3}v_B \]

B1

\[ [v_B = u/6, \quad v_B = u/9, v_C = 2u/9] \]

Use conservation of momentum to find \( k \):

\[ 9mv_B' + kmv_C = 9mv_B \]

\[ 9v_B' + 2kv_B' = 13\cdot5v_B', \quad k = 9/4 \]

M1 A1 3 10
### Question 4

(i) Use conservation of energy at lowest point:
\[ \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga \]
Eliminate \( v^2 \) to find \( R \) \[ R - mg = mv^2/a \]
\[ R = mu^2/a + 3mg = 3\cdot3mg \]

(ii) Use conservation of energy at B to find \( V_B \):
\[ \frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga \sin \theta \]

or
\[ 2\sqrt{(ga/5)} \text{ or } 0.894 \sqrt{(ga)} \]

(A.E.F.)

(iii) Use vertical component \( v_B \) of speed \( V_B \) at B:
\[ v_B = V_B \cos \theta = \frac{1}{4} \sqrt{\frac{15}{g}} \]

\[ V_B = \sqrt{(\frac{3}{4}ga)} \]

Find height \( h \) reached above B:
\[ h = \frac{v_B^2}{2g} \]

Find height \( h \) reached above level of O:
\[ h - a \sin \theta = \frac{3a}{8} - \frac{1}{4}a = a/8 \text{ A.G.} \]

Find MI of components about A:
\[ I = 128 Ma^2 \text{ A.G.} \]

Find total MI about A:
(OR can first find total MI about centre of mass)
State or imply total mass acts at mid-point of AC

\[ I = 128 Ma^2 \text{ A.G.} \]

Use eqn of circular motion to find \( d^2\theta/dt^2 \):
Approximate sin \( \theta \) by \( \theta \) and substitute for I:

\[ T = 2\pi/\omega \text{ with } \omega = \sqrt{(49g/384a)} \]

\[ T = 2\pi \sqrt{(384a/49g)} \text{ or } (16\pi/7) \sqrt{(6a/g)} \text{ or } 17.6 \sqrt{(a/g)} \text{ (A.E.F.)} \]

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| 4 (i)    | Use conservation of energy at lowest point:  
Use \( F = ma \) radially at lowest point:  
Eliminate \( v^2 \) to find \( R \) \[ v^2 = 2\cdot3ga \]: | \[ \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga \]  
\[ R - mg = mv^2/a \]  
\[ R = mu^2/a + 3mg = 3\cdot3mg \] | B1 3 |
| 4 (ii)   | Use conservation of energy at B to find \( V_B \): | \[ \frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga \sin \theta \] | M1A1 |
| 4 (iii)  | Use vertical component \( v_B \) of speed \( V_B \) at B:  
Find height \( h \) reached above B:  
Find height \( h \) reached above level of O: | \[ v_B = V_B \cos \theta = \frac{1}{4} \sqrt{\frac{15}{g}} \]  
\[ V_B = \sqrt{(\frac{3}{4}ga)} \]  
\[ h = \frac{v_B^2}{2g} = \frac{3a}{8} \]  
\[ h - a \sin \theta = \frac{3a}{8} - \frac{1}{4}a = a/8 \text{ A.G.} \] | M1 4 10 |
| 5       | Find MI of components about A:  
(M1 for BC or CD) | Glass \( (3M/5) \{\frac{1}{5}(5a)^2 + 25a^2\} = 20Ma^2 \)  
\[ AB \{\frac{1}{2}(4a)^2 + (4a)^2\} = 64Ma^2/3 \]  
\[ AD \{\frac{1}{2}(3a)^2 + (3a)^2\} = 4Ma^2 \]  
\[ BC \{\frac{1}{2}(3a)^2 + 73a^2\} = 76Ma^{2/3} \]  
\[ CD \{\frac{1}{2}(4a)^2 + 52a^2\} = 172Ma^{2/3} \] | M1A1 A1 8 |
|         | Find total MI about A:  
(OR can first find total MI about centre of mass)  
State or imply total mass acts at mid-point of AC | \[ I = 128 Ma^2 \text{ A.G.} \] | M1 13 |
|         | Use eqn of circular motion to find \( d^2\theta/dt^2 \):  
Approximate sin \( \theta \) by \( \theta \) and substitute for I: | \[ T = 2\pi/\omega \text{ with } \omega = \sqrt{(49g/384a)} \]  
\[ T = 2\pi \sqrt{(384a/49g)} \text{ or } (16\pi/7) \sqrt{(6a/g)} \text{ or } 17.6 \sqrt{(a/g)} \text{ (A.E.F.)} \] | M1A1 A1 5 |
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<td>6</td>
<td>State or find the expected value of $X$: using $p = \frac{1}{4}$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$E(X) = 1/p = 1/\frac{1}{4} = 4$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>$P(X = 4) = (\frac{3}{4})^\frac{1}{4} = 27/256$ or 0.105</td>
<td>M1 A1</td>
<td>2</td>
</tr>
<tr>
<td>S.R. Using $p = \frac{1}{2}$ can earn B0 M1 A0 M0 A0</td>
<td></td>
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<tr>
<td>7</td>
<td>State probability density function of $T$:</td>
<td></td>
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</tr>
<tr>
<td>(i)</td>
<td>$f(t) = 0.001 \exp (-0.001 t)$ ((t \geq 0)) [ = 0 \text{ (otherwise or } t &lt; 0)]</td>
<td>B1</td>
<td>1</td>
</tr>
<tr>
<td>S.R. $1 - e^{-2} = 0.865$ earns B1 only (max 1/3)</td>
<td></td>
<td></td>
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<tr>
<td>State inequality for $t$ (lose A1 if = or $\leq$):</td>
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<tr>
<td>Solve for $t_{max}$:</td>
<td></td>
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<tr>
<td>(Omitting power 10 earns 0/4; using $1 - (\exp (-0.001 t))^{10}$ can earn M1 A0 M1 A0 only)</td>
<td></td>
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</tr>
<tr>
<td>$= 1 - (1 - e^{-2}) = e^{-2}$ or 0.135</td>
<td></td>
<td></td>
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<tr>
<td>$(\exp (-0.001 t))^{10} \geq [or &gt;] 0.9$</td>
<td>M1 A1</td>
<td>3</td>
<td></td>
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<tr>
<td>$t_{max} = (\ln 0.9) / (-0.01) = 10.5$</td>
<td>M1 A1</td>
<td>4</td>
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<td>8</td>
</tr>
<tr>
<td>State hypotheses ( B_0 ) for ( \chi ):</td>
<td></td>
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<tr>
<td>Estimate both popln. variances using two samples:</td>
<td></td>
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</tr>
<tr>
<td>(allow use of biased: ( \sigma_{X,60}^2 = 236 ) or ( 15\cdot36^2 ))</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(allow use of biased: ( \sigma_{Y,50}^2 = 265 ) or ( 16\cdot28^2 ))</td>
<td></td>
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</tr>
<tr>
<td>Estimate population variance for combined sample:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(allow ( \sigma_{X,60}^2/60 + \sigma_{Y,50}^2/50: ) ( 9\cdot233 ) or ( 3\cdot039^2 ))</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Calculate value of ( z ) ((to 2 \text{ d.p., either sign):} )</td>
<td></td>
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</tr>
<tr>
<td>State or use correct tabular ( z – \text{value} ) ((to 2 \text{ d.p.):} )</td>
<td></td>
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<tr>
<td>(or can compare 6 with e.g. ( 2\cdot326 ) ( s = 7\cdot13 ) or ( 7\cdot07 ))</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Correct conclusion ((A.E.F, \checkmark \text{on } z – \text{values):} )</td>
<td></td>
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<tr>
<td>S.R. Assuming equal population variances:</td>
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<tr>
<td>Find pooled estimate of common variance ( s^2 ):</td>
<td></td>
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<tr>
<td>Calculate value of ( z ) ((to 2 \text{ d.p., either sign):} )</td>
<td></td>
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<tr>
<td>Tabular value; conclusion</td>
<td></td>
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\[ H_0: \mu_X = \mu_Y, \quad H_1: \mu_X \neq \mu_Y \]

\[
S_x^2 = \frac{(626220 - 6060^2)/60}{59}
\]

\[
[= 240 \text{ or } 15\cdot49^2]
\]

\[
And \ s_y^2 = \frac{(464500 - 4750^2)/50}{49}
\]

\[
[= 270\cdot4 \text{ or } 16\cdot44^2]
\]

\[
s^2 = \frac{s_x^2}{60} + \frac{s_y^2}{50}
\]

\[
= \frac{9\cdot408}{3\cdot067^2}
\]

\[
z = \frac{(101 - 95)}{s}
\]

\[
= \frac{6\cdot3\cdot067}{1\cdot96} \text{ (or } 1\cdot97)\]

\[
z_{0.99} = 2\cdot326 \text{ or } 2\cdot33 \text{ (allow } 2\cdot36)\]

\[
[\text{Accept } H_0] \text{ Claims are the same} \quad \text{B1}
\]

\[
\text{Hypotheses; Explicit assumption:} \quad \text{B1; B1}
\]

\[
: \quad s^2 = \frac{(626220 - 6060^2)/60 + 464500 - 4750^2/50}{108}
\]

\[
\]

\[
z = \frac{6}{s\sqrt{1/60+1/50}} = 1\cdot97
\]

\[
= \frac{253\cdot8}{1\cdot97^2} \text{ (M1 A1) (A1)}
\]

\[
\text{As above} \quad \text{B1; B1\checkmark}
\]
| 9 | Find expected frequency \( p \): | \( p = 200 \int_2^3 \frac{1}{x \ln 8} \, dx \) |
|   |   | \( = \frac{200}{ \ln 8 } \left[ \ln x \right]_2^3 \) |
|   |   | \( = 200 \times 0.1950 = 39.00 \text{ A.G.} \) |
|   | Find \( q \) by similar method \textit{or} by using total of 200: | \( q = 21.46 \text{ or } 21.45 \) |
|   | State (at least) null hypothesis: | \( H_0: f(x) \text{ fits data} \text{ (A.E.F.)} \) |
|   | Calculate \( \chi^2 \) (to 3 s.f.): | \( \chi^2 = 0.202 + 0.923 + 0.678 + 0.584 \) |
|   |   | \( + 1.134 + 4.134 + 3.644 = 11.3 \) |
|   | State or use correct tabular \( \chi^2 \) value (to 3 s.f.): | \( \chi^6_{0.95} = 12.59 \) |
|   | Valid method for reaching conclusion: | Accept \( H_0 \) if \( \chi^2 \leq \) tabular value |
|   | Conclusion consistent with correct values (A.E.F): | Distribution fits observations |

| 10 | Find correlation coefficient \( r \): | \( r = \frac{73527 - 866 \times 639}{\sqrt{(121276 - 866^2) / 10}} \) |
|    | (A.E.F.; A0 if only 3 s.f. clearly used) | \( = \frac{18189.6 \times 46280.4 \times 15158.9}{0.687} \) |
|    | State both hypotheses (B0 for \( r \) …): | \( H_0: \rho = 0, \ H_1: \rho \neq 0 \) |
|    | State or use correct tabular two-tail \( r \)-value: | \( r_{10,95} = 0.632 \) |
|    | Valid method for reaching conclusion: | Reject \( H_0 \) if \( |r| > \) tabular value |
|    | Correct conclusion (A.E.F, dep *A1, *B1): | There is non-zero correlation |
|    | Calculate gradient \( p \) in \( x - \bar{x} = p(y - \bar{y}) \): | \( p = 18.189.6 / 15158.9 = 1.20 \) |
|    | Find regression line of \( x \) on \( y \): | \( x = 86.6 + 1.20(y - 63.9) \) |
|    |   | \( = 1.20y + 9.92 \) |

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### 11 A

(i) Use Pythagoras to find $AB$:

$$AB = \sqrt{(4a^2 + 12a^2)} = 4a$$

A.G. M1 A1

(ii) Find $\angle SAB$:

$$\angle CAB = \sin^{-1} \frac{2a\sqrt{3}}{4a} \text{ or } \cos^{-1} \frac{2a}{4a}$$

or: $\tan^{-1} \frac{2a\sqrt{3}}{2a}$

$$= 60^\circ \text{ so } \angle SAB = 30^\circ$$

A.G. M1 A1 4

EITHER

Resolve vertically and horizontally, e.g.:

$$\frac{1}{2} N_A + \frac{1}{2} \sqrt{3} N_B + \frac{1}{2} \sqrt{3} F_A = W$$

$(F_A \text{ may be in either direction})$

and $\frac{1}{2} \sqrt{3} N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$

M1 A1

Eliminate $N_B + F_A$ to find $N_A$:

$$N_A = \frac{1}{2} W$$

A.G. A1

OR

Resolve in dirn. $PQ$ to find $N_A$:

$$N_A = \frac{1}{2} W$$

A.G. (M1 A1)

Second resolution, e.g. in dirn. $PS$:

$$N_B + F_A = \frac{1}{2} \sqrt{3} W$$

(A1)

Take moments, e.g. about $A$:

$$\frac{1}{2} \sqrt{3} W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$$

$$= N_B \times 2a$$

M1 A1 A1

Solve to find $N_B$:

$$N_B = \{(7\sqrt{3} - 6)/8\} W$$

M1 A1

Use $N_B$ to find $F_A$:

$$F_A = \sqrt{3} N_A - N_B \text{ or } \frac{1}{2} \sqrt{3} W - N_B$$

$$= \{3(2 - \sqrt{3})/8\} W$$

(A.E.F.) M1 A1 7

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B

Estimate population variance: \( s_p^2 = (236.0 - 42.8^2/8) / 7 \)

(allow biased here: 0.8775 or 0.9367²)

= 351/350 or 1.003 or 1.001²  M1

Find confidence interval (allow z in place of t) e.g.: \( 42.8/8 \pm t \sqrt{(s_p^2/8)} \)  M1

Use correct tabular t-value:

\( t_{7, 0.975} = 2.365 \)  A1

Evaluate C.I. correct to 2 d.p.: 5.35 ± 0.84 or [4.51, 6.19]  A1 4

Formulate inequality for \( k \) (or equality for \( k_{\text{max}} \)):  \( (5.35 - k) / \sqrt{(s_p^2/8)} \geq \) [or >] \( t \)  M1

Use correct tabular t-value:

\( t_{7, 0.9} = 1.415 \)  A1

Solve for \( k_{\text{max}} \) (A0 if = or \( \leq \) was used for \( k \) above): 5.35 - \( k \geq 0.50, k_{\text{max}} = 4.85 \)  A1 3

State hypotheses (B0 for \( \bar{x} \) ...), e.g.:  \( H_0: \mu_P = \mu_Q, H_1: \mu_P > \mu_Q \)  B1

State assumption (A.E.F.): Normal distns. for \( P \) and \( Q \)

\( and \) equal variances  B1

Estimate (pooled) common variance: \( s^2 = (7 \times 1.003 + 11 \times 1.962)/18 \)

= 1.589 or 1.261²  M1  A1

Calculate value of \( t \) (to 3 s.f.):

\( t = (5.35 - 4.60)/(s \sqrt{(1/8 + 1/12)}) \)

= 1.30  M1  A1

Correct conclusion (A.E.F., \( \sqrt{ } \) on \( t \)):  \( t < t_{18, 0.9} = 1.33 \) so \( Q \)'s mean is not less than \( P \)'s  B1 7 14